

# Event Clock Automata: from Theory to Practice\*

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## Abstract

*Event clock automata* (ECA) are a model for *timed languages* that has been introduced by Alur, Fix and Henzinger as an alternative to *timed automata*, with better theoretical properties (for instance, ECA are determinizable while timed automata are not). In this paper, we *revisit* and *extend* the theory of ECA. We first prove that *no finite time abstract language equivalence* exists for ECA, thereby disproving a claim in the original work on ECA. This means in particular that regions *do not form a time abstract bisimulation*. Nevertheless, we show that regions can still be used to build a finite automaton recognizing the *untimed language of an ECA*. Then, we extend the classical notions of *zones* and *DBMs* to let them handle event clocks instead of plain clocks (as in timed automata) by introducing *event zones* and *Event DBMs* (EDBMs). We discuss algorithms to handle event zones represented as EDBMs, as well as (semi-) algorithms based on EDBMs to decide language emptiness of ECA.

## 1 Introduction

*Timed automata* have been introduced by Alur and Dill in the early nineties [2] and are a successful and popular model to reason about *timed behaviors* of computer systems. Where finite automata represent behaviors by finite sequences of actions, timed

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automata define sets of *timed words* (called *timed languages*) that are finite sequences of actions, each paired with a real time stamp. To this end, timed automata extend finite automata with a finite set of real valued clocks, that can be tested and reset with each action of the system. The theory of timed automata is now well developed [1]. The algorithms to analyse timed automata have been implemented in several tools such as Kronos [7] or UppAal (which is increasingly applied in industrial case studies) [4].

Timed automata, however, suffer from certain weaknesses, at least from the theoretical point of view. As a matter of fact, timed automata are *not determinizable* and *cannot be complemented* in general [2]. Intuitively, this stems from the fact that the reset of the clocks cannot be made deterministic wrt the word being read. Indeed, from a given location, there can be two transitions, labeled by the same action  $a$  but different reset sets.

This observation has prompted Alur, Fix and Henzinger to introduce the class of *event clock automata* (ECA for short) [3], as an alternative model for timed languages. Unlike timed automata, ECA force the clock resets to be strongly linked to the occurrences of actions. More precisely, for each action  $a$  of the system, there are two clocks  $\overleftarrow{x}_a$  and  $\overrightarrow{x}_a$  in an ECA:  $\overleftarrow{x}_a$  is the *history clock* of  $a$  and *always records the time elapsed since the last occurrence of  $a$* . Symmetrically,  $\overrightarrow{x}_a$  is the *prophecy clock* for  $a$ , and *always predicts the time distance up to the next occurrence of  $a$* . As a consequence, while history clocks see their values *increase* with time elapsing (like clocks in timed automata do), the values of prophecy clocks *decrease over time*. However, this scheme ensures that the value of any clock is uniquely determined at any point in the timed word being read, no matter what path is being followed in the ECA. A nice consequence of this definition is that ECA are *determinizable* [3]. While the theory of ECA has witnessed some developments [13, 11, 15, 9, 12] since the seminal paper, no tool is available that exploits the full power of event clocks (the only tool we are aware of is TEMPO [14] and it is restricted to *event-recording automata*, i.e. ECA with history clocks only).

In this paper, we revisit and extend the theory of ECA, with the hope to make it more practical and amenable to implementation. A widespread belief [3] about ECA and their analysis is that ECA are similar enough to timed automata that the classical techniques (such as regions, zones or DBMs) developed for them can readily be applied to ECA. The present research, however, highlights *fundamental discrepancies* between timed automata and ECA:

1. First, we show that *there is no finite time abstract language equivalence* on the valuations of *event clocks*, whereas the region equivalence [2] *is* a finite time abstract language equivalence for timed automata. This implies, in particular, that *regions do not form a finite time-abstract bisimulation for ECA*, thereby contradicting a claim found in the original paper on ECA [3].
2. With timed automata, checking language emptiness can be done by building the so-called region automaton [2] which recognizes  $\text{Untime}(L(A))$ , the untimed version of  $A$ 's timed language. A consequence of the surprising result of point 1 is that, for some ECA  $A$ , the *region automaton recognizes a strict subset of  $\text{Untime}(L(A))$* . Thus, the region automaton (as defined in [2]) *is not a sound construction for checking language emptiness of ECA*. We show however that

a slight modification of the original definition (that we call the *existential region automaton*) allows to recover  $\text{Untime}(L(A))$ . Unlike the timed automata case, our proof cannot rely on bisimulation arguments, and requires original techniques.

3. Efficient algorithms to analyze timed automata are best implemented using *zones* [1], that are in turn represented by *DBMs* [10]. Unfortunately, zones and DBMs cannot be directly applied to ECA. Indeed, a zone is, roughly speaking, a conjunction of constraints of the form  $x - y \prec c$ , where  $x, y$  are clocks,  $\prec$  is either  $<$  or  $\leq$  and  $c$  is an integer. This makes sense in the case of timed automata, since the difference of two clock values is an invariant with time elapsing. This is not the case when we consider event clocks, as *prophecy and history clocks evolve in opposite directions with time elapsing*. Thus, we introduce the notions of event-zones and Event DBMs that can handle constraints of the form  $x + y \prec c$ , when  $x$  and  $y$  are of different types.
4. In the case of timed automata two basic, zone-based algorithms for solving language emptiness have been studied: the *forward analysis algorithm* that iteratively computes all the states reachable from the initial state, and the *backward analysis algorithm* that computes all the states that can reach an accepting state. While the former might not terminate in general, the latter is guaranteed to terminate [1]. We show that this is not the case anymore with ECA: *both algorithms might not terminate* again because of event clocks evolving in opposite directions.

These observations reflect the structure of the paper. We close it by discussing the possibility to define *widening operators*, adapted from the *closure by region*, and the *k-approximation* that have been defined for timed automata [6]. The hardest part of this future work will be to obtain a proof of correctness for these operators, since, here again, we will not be able to rely on bisimulation arguments.

## 2 Preliminaries

**Words and timed words** An alphabet  $\Sigma$  is a finite set of symbols. A (finite) *word* is a finite sequence  $w = w_0 w_1 \cdots w_n$  of elements of  $\Sigma$ . We denote the length of  $w$  by  $|w|$ . We denote by  $\Sigma^*$  the set of words over  $\Sigma$ . A *timed word* over  $\Sigma$  is a pair  $\theta = (\tau, w)$  such that  $w$  is a word over  $\Sigma$  and  $\tau = \tau_0 \tau_1 \cdots \tau_{|w|-1}$  is a word over  $\mathbb{R}^{\geq 0}$  with  $\tau_i \leq \tau_{i+1}$  for all  $0 \leq i < |w| - 1$ . We denote by  $\mathbb{T}\Sigma^*$  the set of timed words over  $\Sigma$ . A (timed) *language* is a set of (timed) words. For a timed word  $\theta = (\tau, w)$ , we let  $\text{Untime}(\theta) = w$ . For a timed language  $L$ , we let  $\text{Untime}(L) = \{\text{Untime}(\theta) \mid \theta \in L\}$ .

**Event clocks** Given an alphabet  $\Sigma$ , we define the set of associated *event clocks*  $\mathbb{C}_\Sigma = \mathbb{H}_\Sigma \cup \mathbb{P}_\Sigma$ , where  $\mathbb{H}_\Sigma = \{\overleftarrow{x}_\sigma \mid \sigma \in \Sigma\}$  is the set of *history clocks*, and  $\mathbb{P}_\Sigma = \{\overrightarrow{x}_\sigma \mid \sigma \in \Sigma\}$  is the set of *prophecy clocks*. A *valuation* of a set of clocks is a function  $v : C \rightarrow \mathbb{R}^{\geq 0} \cup \{\perp\}$ , where  $\perp$  means that the clock value is undefined. We denote by  $\mathcal{V}(C)$  the set of all valuations of the clocks in  $C$ . For a valuation  $v \in \mathcal{V}(C)$ , for

all  $x \in \mathbb{H}_\Sigma$ , we let  $\langle v_1(x) \rangle = \lceil v(x) \rceil - v(x)$  and for all  $x \in \mathbb{P}_\Sigma$ , we let  $\langle v(x) \rangle = v(x) - \lfloor v(x) \rfloor$ , where  $\lfloor v(x) \rfloor$  and  $\lceil v(x) \rceil$  denote respectively the largest previous and smallest following integer. We also denote by  $v^\pm$  the valuation s.t.  $v^\pm(x) = v(x)$  for all  $x \in \mathbb{H}_\Sigma$ , and  $v^\pm(x) = -v(x)$  for all  $x \in \mathbb{P}_\Sigma$ .

For all valuation  $v \in \mathcal{V}(C)$  and all  $d \in \mathbb{R}^{\geq 0}$  such that  $v(x) \geq d$  for all  $x \in \mathbb{P}_\Sigma \cap C$ , we define the valuation  $v + d$  obtained from  $v$  by letting  $d$  time units elapse: for all  $x \in \mathbb{H}_\Sigma \cap C$ ,  $(v + d)(x) = v(x) + d$  and for all  $x \in \mathbb{P}_\Sigma \cap C$ ,  $(v + d)(x) = v(x) - d$ , with the convention that  $\perp + d = \perp - d = \perp$ . A valuation is *initial* iff  $v(x) = \perp$  for all  $x \in \mathbb{H}_\Sigma$ , and *final* iff  $v(x) = \perp$  for all  $x \in \mathbb{P}_\Sigma$ . We note  $v[x := c]$  the valuation that matches  $v$  on all its clocks except for  $v(x)$  that equals  $c$ .

An *atomic clock constraint* over  $C \subseteq \mathbb{C}_\Sigma$  is either true or of the form  $x \sim c$ , where  $x \in C$ ,  $c \in \mathbb{N}$  and  $\sim \in \{<, >, =\}$ . A *clock constraint* over  $C$  is a Boolean combination of atomic clock constraints. We denote  $\text{Constr}(C)$  the set of all possible clock constraints over  $C$ . A valuation  $v \in \mathcal{V}(C)$  satisfies a clock constraint  $\psi \in \text{Constr}(C)$ , denoted  $v \models \psi$  according to the following rules:  $v \models \text{true}$ ,  $v \models x \sim c$  iff  $v(x) \sim c$ ,  $v \models \neg\psi$  iff  $v \not\models \psi$ , and  $v \models \psi_1 \wedge \psi_2$  iff  $v \models \psi_1$  and  $v \models \psi_2$ .

**Event-clock automata** An *event-clock automaton*  $A = \langle Q, q_i, \Sigma, \delta, \alpha \rangle$  (ECA for short) is a tuple, where  $Q$  is a finite set of locations,  $q_i \in Q$  is the initial location,  $\Sigma$  is an alphabet,  $\delta \subseteq Q \times \Sigma \times \text{Constr}(\mathbb{C}_\Sigma) \times Q$  is a finite set of edges, and  $\alpha \subseteq Q$  is the set of accepting locations. We additionally require that, for each  $q \in Q$ ,  $\sigma \in \Sigma$ ,  $\delta$  is defined for a finite number of  $\psi \in \text{Constr}(\mathbb{C}_\Sigma)$ . An *extended state* (or simply state) of an ECA  $A = \langle Q, q_i, \Sigma, \delta, \alpha \rangle$  is a pair  $(q, v)$  where  $q \in Q$  is a location, and  $v \in \mathcal{V}(\mathbb{C}_\Sigma)$  is a valuation.

**Runs and accepted language** The semantics of an ECA  $A = \langle Q, q_i, \Sigma, \delta, \alpha \rangle$  is best described by an infinite transition system  $\text{TS}_A = \langle Q^A, Q_i^A, \rightarrow, \alpha^A \rangle$ , where  $Q^A = Q \times \mathcal{V}(\mathbb{C}_\Sigma)$  is the set of extended states of  $A$ ,  $Q_i^A = \{(q_i, v) \mid v \text{ is initial}\}$ ,  $\alpha^A = \{(q, v) \mid q \in \alpha \text{ and } v \text{ is final}\}$ . The transition relation  $\rightarrow \subseteq Q^A \times \mathbb{R}^{\geq 0} \times Q^A \cup Q^A \times \Sigma \times Q^A$  is s.t. (i)  $((q, v), t, (q, v')) \in \rightarrow$  iff  $v' = v + t$  (we denote this by  $(q, v) \xrightarrow{t} (q, v')$ ), and (ii)  $((q, v), \sigma, (q', v')) \in \rightarrow$  iff there is  $(q, \sigma, \psi, q') \in \delta$  and  $\bar{v} \in \mathcal{V}(\mathbb{C}_\Sigma)$  s.t.  $\bar{v}[\bar{x}_\sigma := 0] = v$ ,  $\bar{v}[\bar{x}_\sigma := 0] = v'$  and  $\bar{v} \models \psi$  (we denote this  $(q, v) \xrightarrow{\sigma} (q', v')$ ). We note  $(q, v) \xrightarrow{t, \sigma} (q', v')$  whenever there is  $(q'', v'')$  s.t.  $(q, v) \xrightarrow{t} (q'', v'')$   $\xrightarrow{\sigma} (q', v')$ . Intuitively, this means that an history clock  $\bar{x}_\sigma$  always records the time elapsed since the last occurrence of the corresponding  $\sigma$  event, and that a prophecy clock  $\bar{x}_\sigma$  always predicts the delay up to the next occurrence of  $\sigma$ . Thus, when firing a  $\sigma$ -labeled transition, the guard must be tested against  $\bar{v}$  (as defined above) because it correctly predicts the next occurrence of  $\sigma$  and correctly records its last occurrence (unlike  $v$  and  $v'$ , as  $v(\bar{x}_\sigma) = 0$  and  $v'(\bar{x}_\sigma) = 0$ ).

A sequence  $(q_0, v_0)(t_0, w_0)(q_1, v_1)(t_1, w_1)(q_2, v_2) \cdots (q_n, v_n)$  is a  $(q, v)$ -run of  $A$  on the timed word  $\theta = (\tau, w)$  iff:  $(q_0, v_0) = (q, v)$ ,  $t_0 = \tau_0$ , for any  $1 \leq i \leq n-1$ :  $t_i = \tau_i - \tau_{i-1}$ , and for any  $0 \leq i \leq n-1$ :  $(q_i, v_i) \xrightarrow{t_i, w_i} (q_{i+1}, v_{i+1})$ . A  $(q, v)$ -run is *initialized* iff  $(q, v) \in Q_i^A$  (in this case, we simply call it a run). A  $(q, v)$ -run on  $\theta$ , ending in  $(q_n, v_n)$  is *accepting* iff  $(q_n, v_n) \in \alpha^A$ . In this case, we say that the run

accepts  $\theta$ . For an ECA  $A$  and an extended state  $(q, v)$  of  $A$ , we denote by  $L(A, (q, v))$  the set of timed words accepted by a  $(q, v)$ -run of  $A$ , and by  $L(A)$  the set of timed words accepted by an initialized run of  $A$ .

### 3 Equivalence relations for event-clocks

A classical technique to analyze timed transition systems is to define *time abstract equivalence relations* on the set of states, and to reason on the *quotient* transition system. In the case of *timed automata*, a fundamental concept is the *region equivalence* [2], which is a *finite time-abstract* bisimulation, and allows to decide properties of timed automata such as reachability. Contrary to a widespread belief [3], we show that the class of ECA does not benefit of these properties, as ECA **admit no finite time-abstract language equivalence**.

**Time-abstract equivalence relations** Let  $\mathcal{C}$  be a class of ECA, all sharing the same alphabet  $\Sigma$ . We recall three equivalence notions on event clock valuations:

- $\lesssim \subseteq \mathcal{V}(\mathbb{C}_\Sigma) \times \mathcal{V}(\mathbb{C}_\Sigma)$  is a *time abstract simulation relation* for the class  $\mathcal{C}$  iff, for all  $\mathcal{A} \in \mathcal{C}$ , for all location  $q$  of  $\mathcal{A}$ , for all  $(v_1, v_2) \in \lesssim$ , for all  $t_1 \in \mathbb{R}^{\geq 0}$ , for all  $a \in \Sigma$ :  $(q, v_1) \xrightarrow{t_1, a} (q', v'_1)$  implies that there exists  $t_2 \in \mathbb{R}^{\geq 0}$  s.t.  $(q, v_2) \xrightarrow{t_2, a} (q', v'_2)$  and  $v'_1 \lesssim v'_2$ . In this case, we say that  $v_2$  *simulates*  $v_1$ . Finally,  $\simeq \subseteq \mathcal{V}(\mathbb{C}_\Sigma) \times \mathcal{V}(\mathbb{C}_\Sigma)$  is a *time abstract simulation equivalence* iff there exists a time abstract simulation relation  $\lesssim$  s.t.  $\simeq = \{(v_1, v_2) \mid v_1 \lesssim v_2 \text{ and } v_2 \lesssim v_1\}$
- $\sim$  is a *time abstract bisimulation equivalence* for the class  $\mathcal{C}$  iff it is a *symmetric* time abstract simulation for the class  $\mathcal{C}$ .
- $\approx_L \subseteq \mathcal{V}(\mathbb{C}_\Sigma) \times \mathcal{V}(\mathbb{C}_\Sigma)$  is a *time abstract language equivalence* for the class  $\mathcal{C}$  iff for all  $\mathcal{A} \in \mathcal{C}$ , for all location  $q$  of  $\mathcal{A}$ , for all  $(v_1, v_2) \in \approx_L$ :  $\text{Untime}(L(q, v_1)) = \text{Untime}(L(q, v_2))$

We say that an equivalence relation is *finite* iff it is of finite index. Clearly, any time abstract bisimulation is a time abstract simulation equivalence, and any time abstract simulation equivalence is a time abstract language equivalence. We prove the absence of *finite* time abstract language equivalence for ECA, thanks to  $A_{\text{inf}}$  depicted in Fig. 1:

**Proposition 1.** *There is no finite time abstract language equivalence for ECA.*

*Proof.* Let us assume that  $\approx_L$  is a time abstract language equivalence on the class of ECA. We will show, thanks to  $A_{\text{inf}}$ , that  $\approx_L$  has necessarily *infinitely* many equivalence classes.

For any  $n \in \mathbb{N}$ , let  $v^n$  denote the *initial* valuation of  $\mathbb{C}_{\{a, b\}}$  s.t.  $v^n(\vec{x}_a) = n$  and  $v^n(\vec{x}_b) = 0$ , and let  $\theta^n$  denote the timed word  $(b, 0)(b, 1)(b, 2) \cdots (b, n-1)(a, n)$ . Observe that, for any  $n \geq 0$ , there is only one run of  $A_{\text{inf}}$  starting in  $(q_0, v^n)$  and this run accepts  $\theta^n$ . Hence, for any  $n \geq 0$ :  $\text{Untime}(L(A, (q_0, v^n))) = \text{Untime}(\{\theta^n\}) = a^n b$ .

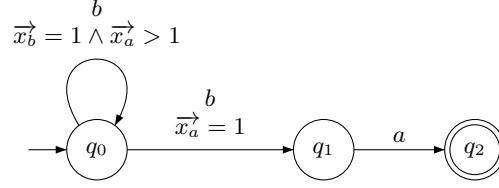


Figure 1: The automaton  $A_{\text{inf}}$

Now, let  $j, k$  be two natural values with  $j \neq k$ . Let  $s^j = (q_0, v^j)$  and  $s^k = (q_0, v^k)$ . Clearly,  $v^j \not\approx_L v^k$  since  $\text{Untime}(\mathbb{L}(A, s^j)) \neq \text{Untime}(\mathbb{L}(A_{\text{inf}}, s^k))$ . Since this is true for infinitely many pairs  $(v^j, v^k)$ ,  $\approx_L$  has necessarily an infinite number of equivalence classes. Thus, there is no *finite* time abstract language equivalence on the class of ECA.  $\square$

**Corollary 1.** *There is no finite time abstract language equivalence, no finite time abstract simulation equivalence and no finite time abstract bisimulation for ECA.*

## 4 Regions and event clocks

For the class of timed automata, the *region equivalence* has been shown to be a *finite time-abstract bisimulation*, which is used to build the so-called *region automaton*, a finite-state automaton recognizing  $\text{Untime}(L(A))$  for all timed automata  $A$  [2]. Corollary 1 tells us that regions are not a time-abstract bisimulation for ECA (contrary to what was claimed in [3]). Let us show that we can nevertheless rely on the notion of region to build a finite automaton recognizing  $\text{Untime}(L(A))$  for all ECA  $A$ .

**Regions** Let us fix a set of clocks  $C \subseteq \mathbb{C}_\Sigma$  and a constant  $c_{\text{max}} \in \mathbb{N}$ . We first recall two region equivalences from the literature. The former, denoted  $\approx_{c_{\text{max}}}$ , is the classical Alur-Dill region equivalence for timed automata [2] while the latter (denoted  $\approx_{c_{\text{max}}}^\angle$ ) is adapted from Bouyer [6] and refines the former:

- For any  $v_1, v_2 \in \mathcal{V}(C)$ :  $v_1 \approx_{c_{\text{max}}} v_2$  iff:
  - (C1) for all  $x \in C$ ,  $v_1(x) = \perp$  iff  $v_2(x) = \perp$ ,
  - (C2) for all  $x \in C$ : either  $v_1(x) > c_{\text{max}}$  and  $v_2(x) > c_{\text{max}}$ , or  $\lceil v_1(x) \rceil = \lceil v_2(x) \rceil$  and  $\lfloor v_1(x) \rfloor = \lfloor v_2(x) \rfloor$ ,
  - (C3) for all  $x_1, x_2 \in C$  s.t.  $v_1(x_1) \leq c_{\text{max}}$  and  $v_1(x_2) \leq c_{\text{max}}$ :  $\langle v_1(x_1) \rangle \leq \langle v_1(x_2) \rangle$  if and only if  $\langle v_2(x_1) \rangle \leq \langle v_2(x_2) \rangle$ .
- For all  $v_1, v_2 \in \mathcal{V}(C)$ :  $v_1 \approx_{c_{\text{max}}}^\angle v_2$  iff:  $v_1 \approx_{c_{\text{max}}} v_2$  and:

(C4) For all  $x_1, x_2 \in C$  s.t.  $v_1(x_1) > cmax$  or  $v_1(x_2) > cmax$ : either we have  $|v_1^\pm(x_1) - v_1^\pm(x_2)| > 2 \cdot cmax$  and  $|v_2^\pm(x_1) - v_2^\pm(x_2)| > 2 \cdot cmax$ ; or we have  $\lfloor v_1^\pm(x_1) - v_1^\pm(x_2) \rfloor = \lfloor v_2^\pm(x_1) - v_2^\pm(x_2) \rfloor$  and  $\lceil v_1^\pm(x_1) - v_1^\pm(x_2) \rceil = \lceil v_2^\pm(x_1) - v_2^\pm(x_2) \rceil$ .

Equivalence classes of both  $\approx_{cmax}$  and  $\approx_{cmax}^\angle$  are called *regions*. We denote by  $\text{Reg}(C, cmax)$  and  $\text{Reg}^\angle(C, cmax)$  the set of regions of  $\approx_{cmax}$  and  $\approx_{cmax}^\angle$  respectively. Fig. 2 (a), (b) and (c) illustrate these two notions. Comparing (a) and (b) clearly shows how  $\approx_{cmax}^\angle$  refines  $\approx_{cmax}$  by introducing diagonal constraints between clocks larger than  $cmax$ . Moreover, (c) shows why we need to rely on  $v_1^\pm$  and  $v_2^\pm$  in C4: in this case,  $C$  contains an history and a prophecy clock that evolve in opposite directions with time elapsing. Thus, their *sum* remains constant over time (hence the  $2 \cdot cmax$  in C4).

Observe that, for any  $cmax$ , and for any finite set of clocks  $C$ ,  $\text{Reg}(C, cmax)$  and  $\text{Reg}^\angle(C, cmax)$  are *finite* sets. A region  $r$  on set of clocks  $C$  is *initial* (resp. *final*) iff it contains only initial (final) valuations.

**Regions are not a language equivalence** Since both notions of regions defined above are finite, Corollary 1 implies that they cannot form a language equivalence for ECA. Let us explain intuitively why it is not the case. Consider  $\text{Reg}(\mathbb{P}_{\{a,b\}}, 1)$  and the two valuations  $v_1$  and  $v_2$  in Fig. 2 (a). Clearly,  $v_1$  can reach the region where  $\vec{x}_a = 1$  and  $\vec{x}_b > 1$ , while  $v_2$  cannot. Conversely,  $v_2$  can reach  $\vec{x}_a > 1$  and  $\vec{x}_b = 1$  but  $v_1$  cannot. It is easy to build an ECA with  $cmax = 1$  that distinguishes between those two cases and accepts different words. Then, consider  $\text{Reg}^\angle(\mathbb{P}_{\{a,b\}}, 1)$  and the valuations  $v^3$  and  $v^4$  (not shown in the figure) s.t.  $v^3(\vec{x}_b) = v^4(\vec{x}_b) = 1$ ,  $v^3(\vec{x}_a) = 4$  and  $v^4(\vec{x}_a) = 5$ . It is easy to see that for  $A_{\text{inf}}$  in Fig. 1:  $\text{Untime}(L(A_{\text{inf}}, (q_0, v^3))) = \{bbba\} \neq \{bbba\} = \text{Untime}(L(A_{\text{inf}}, (q_0, v^4)))$ , although  $v^3$  and  $v^4$  belong to the same region. Indeed, from  $v^3$ , the  $(q_0, q_0)$  loop can be fired 3 times before we reach  $\vec{x}_a = 1$  and the  $(q_0, q_1)$  edge can be fired. However, the  $(q_0, q_0)$  loop has to be fired 4 times from  $v^4$  before we reach  $\vec{x}_a = 1$  and the  $(q_0, q_1)$  edge can be fired. Remark that these are essentially the same arguments as in the proof of Proposition 1. These two examples illustrate the issue with *prophecy clocks* and regions. Roughly speaking, to keep the set of regions finite, valuations where the clocks are *too large* (for instance,  $> cmax$  in the case of  $\text{Reg}(C, cmax)$ ) belong to the same region. This is not a problem for history clocks as an history clock larger than  $cmax$  remains over  $cmax$  with time elapsing. This is not the case for prophecy clocks whose values *decrease with time elapsing*: eventually, those clocks reach a value  $\leq cmax$ , but the region equivalence is too coarse to allow to predict the region they reach.

**Region automata** Let us now consider the consequence of Corollary 1 on the notion of region automaton. We first define two variants of the region automaton:

**Definition 1.** Let  $A = \langle Q, q_i, \Sigma, \delta, \alpha \rangle$  and  $\mathcal{R}$  be a set of regions on  $\mathcal{V}(\mathbb{C}_\Sigma)$ . Then, the *existential* (resp. *universal*)  $\mathcal{R}$ -region automaton of  $A$  is the finite transition system  $RA(\exists, \mathcal{R}, A)$  (resp.  $RA(\forall, \mathcal{R}, A)$ ) defined by  $\langle Q^R, Q_i^R, \Sigma, \delta^R, \alpha^R \rangle$  s.t.:

1.  $Q^R = Q \times \mathcal{R}$

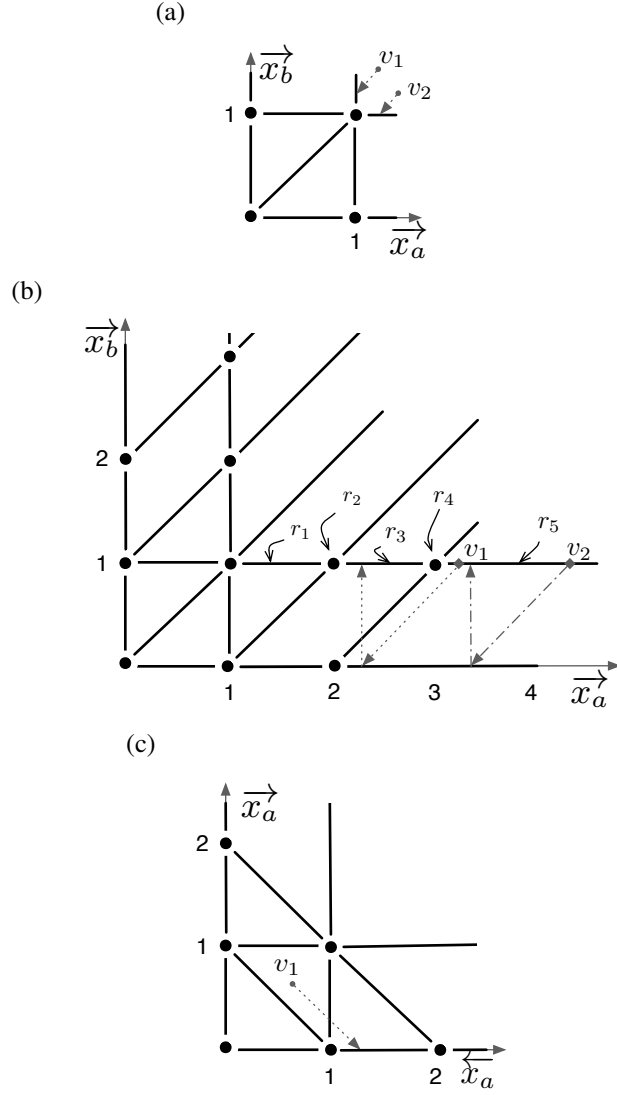


Figure 2: The sets of regions (a)  $\text{Reg}(\mathbb{P}_{\{a,b\}}, 1)$ , (b)  $\text{Reg}^{\angle}(\mathbb{P}_{\{a,b\}}, 1)$  and (c)  $\text{Reg}^{\angle}(\mathbb{C}_{\{a\}}, 1)$ . Dotted arrows show the trajectories followed by the valuations with time elapsing. Curved arrows are used to refer to selected regions.



2.  $Q_i^R = \{(q_i, r) \mid r \text{ is an initial region}\}$
3.  $\delta^R \subseteq Q^R \times \Sigma \times Q^R$  is s.t.  $((q_1, r_1), a, (q_2, r_2)) \in \delta$  iff **there exists a valuation** (resp. **for all valuations**)  $v_1 \in r_1$ , there exists a time delay  $t \in \mathbb{R}^{\geq 0}$  and a valuation  $v_2 \in r_2$  s.t.  $(q_1, v_1) \xrightarrow{t, a} (q_2, v_2)$ .
4.  $\alpha^R = \{(q, r) \mid q \in \alpha \text{ and } r \text{ is a final region}\}$

Let  $R = \langle Q^R, Q_i^R, \Sigma, \delta^R, \alpha^R \rangle$  be a region automaton and  $w$  be an (untimed) word over  $\Sigma$ . A *run* of  $R$  on  $w = w_0 w_1 \dots w_n$  is a finite sequence  $(q_0, r_0)(q_1, r_1) \dots (q_{n+1}, r_{n+1})$  of states of  $R$  such that:  $(q_0, r_0) \in Q_i^R$  and such that: for all  $0 \leq i \leq n$ :  $((q_i, r_i), w_i, (q_{i+1}, r_{i+1})) \in \delta^R$ . Such a run is *accepting* iff  $(q_{n+1}, r_{n+1}) \in \alpha^R$  (in that case, we say that  $w$  is accepted by  $R$ ). The language  $L(R)$  of  $R$  is the set of all untimed words accepted by  $R$ .

Let  $A$  be an ECA with alphabet  $\Sigma$  and maximal constant  $cmax$ . If we adapt and apply the notion of region automaton, as defined for timed automata [2], to  $A$  we obtain  $RA(\forall, \text{Reg}(\mathbb{C}_\Sigma, cmax), A)$ . To alleviate notations, we denote it by  $\text{RegAut}_\forall(A)$ . In the rest of the paper, we also consider three other variants: (i)  $\text{RegAut}_\forall^\prec(A) = RA(\forall, \text{Reg}^\prec(\mathbb{C}_\Sigma, cmax), A)$ , (ii)  $\text{RegAut}_\exists(A) = RA(\exists, \text{Reg}(\mathbb{C}_\Sigma, cmax), A)$  and (iii)  $\text{RegAut}_\exists^\prec(A) = RA(\exists, \text{Reg}^\prec(\mathbb{C}_\Sigma, cmax), A)$ . Observe that, for timed automata, all these automata coincide, and thus accept the untimed language (this can be proved by a bisimulation argument) [2]. Let us see how these results adapt (or not) to ECA.

**Recognized language of universal region automata** Let us show that, in general *universal region automata do not recognize the untimed language of the ECA*.

**Lemma 1.** *There is an ECA  $A$  such that  $L(\text{RegAut}_\forall(A)) \subsetneq \text{Untime}(L(A))$  and such that  $L(\text{RegAut}_\forall^\prec(A)) \subsetneq \text{Untime}(L(A))$ .*

*Proof.* Consider the automaton  $A_{\text{inf}}$  in Fig. 1, with  $cmax = 1$ . Assume there is, in  $\text{RegAut}_\forall(A_{\text{inf}})$ , an edge of the form  $((q_0, r), b, (q_0, r'))$ , where  $r$  is initial. By the guard of the  $(q_0, q_0)$  loop,  $r'$  is a region s.t. for all  $v \in r$ :  $v(\vec{x}_b) = 1$  and  $v(\vec{x}_a) > 1$ . To fire the  $(q_0, q_0)$  loop again, we need to let time elapse up to the point where  $\vec{x}_b = 0$ . Then consider two valuations  $v$  and  $v'$  s.t.  $v(\vec{x}_b) = v'(\vec{x}_b) = 1$ ,  $v(\vec{x}_a) = 1.1$  and  $v'(\vec{x}_a) = 2.1$ . Clearly,  $\{v, v'\} \subseteq r'$ . However, firing the  $(q_0, q_0)$  loop from  $(q_0, v)$  leads to  $(q_0, v'')$ , with  $v''(\vec{x}_a) = 0.1$ , and firing the same  $(q_0, q_0)$  loop from  $(q_0, v')$  leads to  $(q_0, v''')$  with  $v'''(\vec{x}_a) = 1.1$ . Thus,  $v''$  and  $v'''$  do not belong to the same region. Since we are considering a *universal* automaton, we conclude that there is no edge of the form  $((q_0, r'), b, (q_0, r''))$ . Hence,  $\text{RegAut}_\forall(A_{\text{inf}})$  cannot recognize an arbitrary number of  $b$ 's from any of its initial states, and thus,  $L(\text{RegAut}_\forall(A_{\text{inf}})) \subsetneq \text{Untime}(L(A_{\text{inf}}))$ .

For the second case, we consider Fig. 2 (b) that depicts the projection of the set of regions used to build  $\text{RegAut}_\forall^\prec(A_{\text{inf}})$  on the clocks  $\{\vec{x}_a, \vec{x}_b\}$  (remark that we can restrict our reasoning to this projection, since the other clocks are never tested in  $A_{\text{inf}}$ ). Assume there is, in  $\text{RegAut}_\forall^\prec(A_{\text{inf}})$ , an edge of the form  $((q_0, r), b, (q_0, r'))$  where  $r$  is initial. This implies that  $r' \in \{r_1, \dots, r_5\}$  (we refer to the names in Fig. 2), because of the guard of the  $(q_0, q_0)$  loop. Since  $\text{Untime}(L(A_{\text{inf}})) = \{b^n a \mid n \geq 1\}$ , it must be possible to accept an arbitrary number of  $b$ 's from one of the  $(q_0, r')$ . Let us

show that it is not the case. From  $r_3$  and  $r_4$  we have edges  $((q_0, r_3), b, (q_0, r_1))$  and  $((q_0, r_4), b, (q_0, r_2))$ . However, there is no valuation  $v \in r_1 \cup r_2$  s.t.  $(v + t)(\vec{x}_b) = 0$  and  $(v + t)(\vec{x}_a) > 1$  for some  $t$ . Thus, there is, in  $\text{RegAut}_\forall^\angle(A_{\text{inf}})$ , no edge of the form  $((q_0, r), b, (q_0, r'))$  when  $r \in r_1, r_2$ . Finally, there is no edge of the form  $((q_0, r_5), b, (q_0, r))$  because *some* valuations of  $r_5$  (such as  $v_1$ ) will reach  $r_3$  and *some others* (such as  $v_2$ ) will stay in  $r_5$  after the firing of the loop. Since we consider a *universal* automaton,  $(q_0, r_5)$  has no successor.  $\square$

**Recognized language of existential region automata** Fortunately, the definition of *existential region automaton* allows us to recover a finite transition system recognizing exactly  $\text{Uptime}(L(A))$ , for all ECA  $A$ . Remark that our construction is *direct*, contrary to the original construction [3] that consists in first translating the ECA into a non-deterministic timed automaton recognising the same timed language but with an increased number of clocks compared to the original ECA, and then computing the region automaton of this timed automaton. Moreover, the proof we are about to present cannot invoke the fact that regions form a time-abstract bisimulation, as it is the case for timed automata, and we thus need to rely on different proof techniques. Actually, we will show that:

$$\text{Uptime}(L(A)) \subseteq L(\text{RegAut}_\exists^\angle(A)) \subseteq L(\text{RegAut}_\exists(A)) \subseteq \text{Uptime}(L(A))$$

The two leftmost inequalities are easily established by the following reasonings. Let  $(q_0, v_0)(t_0, w_0)(q_1, v_1)(t_1, w_1) \cdots (q_n, v_n)$  be an accepting run of  $A$  on  $\theta = (\tau, w)$ . Thus,  $\theta \in L(A)$ . For all  $0 \leq i \leq n$  let  $r_i$  be the (unique) region containing  $v_i$ . Then, by definition of  $\text{RegAut}_\exists^\angle(A)$ ,  $(q_0, r_0)w_0(q_1, r_1)w_1 \cdots (q_n, r_n)$  is an accepting run of  $\text{RegAut}_\exists^\angle(A)$  on  $w = \text{Uptime}(\theta)$ . Hence  $\text{Uptime}(L(A)) \subseteq L(\text{RegAut}_\exists^\angle(A))$ . Second, since  $\approx_{\text{cmax}}^\angle$  refines  $\approx_{\text{cmax}}$ , each accepting run  $(q_0, r_0)w_0(q_1, r_1)w_1 \cdots (q_n, r_n)$  in  $\text{RegAut}_\exists^\angle(A)$  corresponds to an accepting run  $(q_0, r'_0)w_0(q_1, r'_1)w_1 \cdots (q_n, r'_n)$  in  $\text{RegAut}_\exists(A)$ , where for any  $0 \leq i \leq n$ ,  $r'_i$  is the (unique) region of  $\text{Reg}(\mathbb{C}_\Sigma, \text{cmax})$  that contains  $r_i$ . Hence,  $L(\text{RegAut}_\exists^\angle(A)) \subseteq L(\text{RegAut}_\exists(A))$ .

To establish  $L(\text{RegAut}_\exists(A)) \subseteq \text{Uptime}(L(A))$  we need to rely on the notion of *weak time successor*. The set of *weak time successors* of  $v$  by  $t$  time units is:

$$v +_w t = \left\{ v' \mid \forall x : \begin{array}{l} (x \in \mathbb{P}_\Sigma \text{ and } v(x) > \text{cmax}) \text{ implies } v'(x) > \text{cmax} - t \\ \text{and} \\ (x \notin \mathbb{P}_\Sigma \text{ or } v(x) \leq \text{cmax} \text{ or } v(x) = \perp) \text{ implies } v'(x) = (v + t)(x) \end{array} \right\}$$

As can be seen, weak time successors introduce non-determinism on prophecy clocks that are larger than  $\text{cmax}$ . So,  $v +_w t$  is a *set* of valuations. Let  $q$  be a location of an ECA. We write  $(q, v) \xrightarrow{t}_w (q, v')$  whenever  $v' \in (v +_w t)$ . Then, a sequence  $(q_0, v_0)(t_0, w_0)(q_1, v_1)(t_1, w_1)(q_2, v_2) \cdots (q_n, v_n)$  is an initialized *weak run*, on  $\theta = (\tau, w)$ , of an ECA  $A = \langle Q, q_i, \Sigma, \delta, \alpha \rangle$  iff  $q_0 = q_i$ ,  $v_0$  is initial,  $t_0 = \tau_0$ , for any  $1 \leq i \leq n - 1$ :  $t_i = \tau_i - \tau_{i-1}$ , and for any  $0 \leq i \leq n - 1$ : there is  $(q'_i, v'_i)$  s.t.  $(q_i, v_i) \xrightarrow{t_i}_w (q'_i, v'_i) \xrightarrow{w_i} (q_{i+1}, v_{i+1})$ . A weak run is accepting iff  $q_n \in \alpha$  and  $v_n$  is final. The weak language  $\text{wL}(A)$  of  $A$  is the set of all timed words  $\theta$  s.t. there is an accepting weak run on  $\theta$ . Clearly,  $L(A) \subseteq \text{wL}(A)$  as every run is also a weak run.

However, the converse also holds, since the non-determinism appears only on clocks larger than  $cmax$ , which the automaton cannot distinguish:

**Proposition 2.** *For any ECA  $A$ :  $L(A) = wL(A)$ .*

*Proof.* Since, by definition, every run is a weak run,  $L(A) \subseteq wL(A)$ . Let us show that  $L(A) \supseteq wL(A)$ . Let  $\theta = (\tau_0, w_0) \cdots (\tau_n, w_n)$  be a timed word in  $wL(A)$ , and let  $(q_0, v_0) \xrightarrow{t_0}_w (q_0, v'_0) \xrightarrow{w_0} (q_1, v_1) \cdots (q_{n+1}, v_{n+1})$  be the corresponding accepting weak run of  $A$ . For any  $0 \leq i \leq n$ , we build  $\bar{v}_i$  as follows. For any  $x$  s.t.  $x \in \mathbb{P}_\Sigma$  and  $v'_i(x) > cmax$ , let  $k > i$  be the least position s.t.  $v'_k(x) \leq cmax$ . Remark that such a position always exists in an *accepting* run (recall that if a letter is never to be seen again, its valuation must be set to  $\perp$ ). Then, we let  $\bar{v}_i(x) = v'_k(x) + \sum_{j=i+1}^k t_j$ . Otherwise, we let  $\bar{v}_i(x) = v'_i(x)$ . Remark that  $v'_i$  and  $\bar{v}_i$  differ only on prophecy clocks larger than  $cmax$ , and that  $v'_i(x) > cmax$  iff  $\bar{v}_i(x) > cmax$  for any  $i$  and  $x$ . Moreover, the definition of the sequence of  $\bar{v}_i$  clearly respects the definition of time successor. We further define  $\tilde{v}_i$  for all  $i$  as follows:  $\tilde{v}_i(x) = t_i + \bar{v}_i(x)$  for all  $x \in \mathbb{P}_\Sigma$  s.t.  $v_i(x) > cmax$  and  $\tilde{v}_i(x) = v_i(x)$  otherwise. Hence, it can be checked that for all  $0 \leq i \leq n$ ,  $(q_i, \tilde{v}_i) \xrightarrow{t_i} (q_i, \bar{v}_i) \xrightarrow{w_i} (q_{i+1}, \tilde{v}_{i+1})$ , and so that  $(q_0, \tilde{v}_0) \xrightarrow{t_0} (q_0, \bar{v}_0) \xrightarrow{w_0} (q_1, \tilde{v}_1) \xrightarrow{t_1} (q_1, \bar{v}_1) \cdots (q_{n+1}, \tilde{v}_{n+1})$ . Moreover,  $\tilde{v}_{n+1}(x) = v_{n+1}(x) = \perp$  for all  $x \in \mathbb{P}_\Sigma$ . Thus,  $\theta \in L(A)$  and thus,  $L(A) \supseteq wL(A)$ .  $\square$

Then, we prove that *weak time successors* enjoy a property which is reminiscent of time abstract bisimulation. This allows to establish Theorem 1.

**Lemma 2.** *Let  $C$  be a set of clocks and let  $cmax$  be a natural constant. For any  $v_1, v_2 \in \mathcal{V}(C)$  s.t.  $v_1 \approx_{cmax} v_2$ , for any  $t_1 \in \mathbb{R}^{\geq 0}$ , there exist  $t_2$  and  $v' \in (v_2 +_w t_2)$  s.t.  $v_1 + t_1 \approx_{cmax} v'$ .*

*Proof.* The cases where  $v_1 \approx_{cmax} v_1 + t_1$  are trivial. We first restrict ourselves to the case where  $v_1$  and  $v_1 + t_1$  belong to adjacent regions, that is:

$$\exists 0 < t \leq t_1 : \left( \begin{array}{c} \forall 0 \leq t' \leq t : v_1 + t' \approx_{cmax} v_1 \\ \text{and} \\ \forall t < t' \leq t_1 : v_1 + t' \approx_{cmax} v_1 + t_1 \end{array} \right) \quad (1)$$

Let us now show how to chose  $t_2$ . Let  $C_v^0$  denote the set of clocks  $x$  s.t.  $\langle v(x) \rangle = 0$ . Under the hypothesis (1), we have to consider two cases:

1. Either  $C_{v_1}^0 = \emptyset$  and  $C_{v_1+t_1}^0 \neq \emptyset$ . In that case, let  $x$  be a clock in  $C_{v_1+t_1}^0$ . We let  $t_2 = \langle v_2(x) \rangle$
2. Or  $C_{v_1}^0 \neq \emptyset$  and  $C_{v_1+t_1}^0 = \emptyset$ . In that case, we need to consider two sub-cases. If there is  $x$  s.t.  $\langle v_2(x) \rangle \neq 0$ , we let  $t_2$  be a value s.t.  $0 < t_2 < \min\{\langle v_2(x) \rangle \mid \langle v_2(x) \rangle \neq 0\}$ . Otherwise, all the clocks in  $v_2$  have a null fractional part, and we can take any delay  $< 1$  for  $t_2$ : we let  $t_2 = 0.1$ .

Now, let us show that there exists  $v \in v_2 +_w t_2$  s.t.  $v \approx_{cmax} v_1 + t_1$ . For that purpose, we first build a valuation  $v_3$  as follows. For any history clock  $x$ , we let  $v_3(x) = v_2(x)$ . For all prophecy clocks  $x$  s.t.  $v_2(x) \leq cmax$ , or  $v_2(x) = \perp$ , we let  $v_3(x) = v_2(x)$  too. For all prophecy clocks  $x$  s.t.  $v_2(x) > cmax$  (and thus  $v_1(x) > cmax$  since  $v_1 \approx_{cmax} v_2$ ), we consider two cases. Either  $(v_1 + t_1)(x) > cmax$ . In that case we let  $v_3(x) = cmax + t_2 + 1$ . Or  $(v_1 + t_1)(x) = cmax$ . In that case we let  $v_3(x) = cmax + t_2$ . Remark that the case  $(v_1 + t_1)(x) < cmax$  is not possible since we have assumed that  $v_1(x) > cmax$  and that  $v_1$  and  $v_1 + t_1$  are in adjacent regions.

We now let  $v' = v_3 + t_2$ . It is easy to check that  $v' \approx_{cmax} (v_1 + t_1)$ . Moreover,  $v' \in (v_2 +_w t_2)$ , since  $v_3$  has been obtained from  $v_2$  by replacing values larger than  $cmax$  by other values larger than  $cmax$ .

To conclude, observe that if  $v_3 \in (v_2 +_w t_2)$  and  $v_2 \in (v_1 +_w t_1)$ , then  $v_3 \in (v_1 +_w (t_1 + t_2))$ . This allows to handle the case where  $v_1$  and  $v_1 + t_1$  are not in adjacent regions: by decomposing  $t_1$  into a sequence  $t'_1, t'_2, \dots, t'_n$  s.t.  $t_1 = t'_1 + t'_2 + \dots + t'_n$ , and for all  $1 \leq i < n$ ,  $v_1 + \sum_{j=1}^i t'_j$  and  $v_1 + \sum_{j=1}^{i+1} t'_j$  are in adjacent regions. Then, applying the reasoning above, we get a sequence  $t''_1, \dots, t''_n$  of time delays and a sequence  $v'_0, v'_1, \dots, v'_n$  of valuations s.t.  $v'_0 = v_2$ , for all  $0 \leq i < n$ ,  $v'_{i+1} \in v'_i +_w t''_i$  and  $v'_{i+1} \approx_{cmax} v_1 + \sum_{j=1}^{i+1} t'_j$ . Thus,  $v'_n \in v_2 +_w \sum_{j=1}^n t''_j$  and  $v'_n \approx_{cmax} v_1 + \sum_{j=1}^n t'_j = v_1 + t_1$ .  $\square$

We can now prove that:

**Theorem 1.** For any ECA  $A = (\Sigma, Q, q_i, \delta, \alpha)$ :  $L(\text{RegAut}_{\exists}(A)) \subseteq \text{Untime}(L(A))$ .

*Proof.* Let  $(q_0, r_0) \xrightarrow{w_0} (q_1, r_1) \xrightarrow{w_1} \dots \xrightarrow{w_{n-1}} (q_n, r_n)$  be an accepting run of  $\text{RegAut}_{\exists}(A)$ . Let us build, inductively a sequence  $\bar{t}_0, \bar{t}_1, \dots, \bar{t}_{n-1}$  of time delays and a sequence  $\bar{v}_0, \bar{v}_1, \dots, \bar{v}_n$  of valuations s.t.  $\bar{v}_i \in r_i$  for all  $0 \leq i \leq n$ . This will allow us to obtain an accepting weak run of  $A$ . For the base case, we let  $\bar{v}_0$  be a valuation from  $r_0$  and we let  $\bar{v}_1$  and  $\bar{t}_0$  be s.t.  $\bar{v}_0 \xrightarrow{\bar{t}_0, w_0} \bar{v}_1$  with  $\bar{v}_1 \in r_1$ . Such  $\bar{v}_1$  and  $\bar{t}_0$  are guaranteed to exist by definition of the region automaton, and since  $(q_0, r_0) \xrightarrow{w_0} (q_1, r_1)$  in this region automaton. For the inductive case, we consider  $i$  with  $2 \leq i \leq n$  and assume that  $\bar{v}_{i-1}$  has been defined and is in  $r_{i-1}$ . Let us show how to build  $\bar{t}_{i-1}$  and  $\bar{v}_i$ . Since  $r_{i-1} \xrightarrow{w_{i-1}} r_i$  in the region automaton, there are  $v_i \in r_i$ ,  $v_{i-1} \in r_{i-1}$ , and  $t_{i-1}$  s.t.  $v_{i-1} \xrightarrow{t_{i-1}} v_{i-1} + t_{i-1} \xrightarrow{w_{i-1}} v_i$ . Let  $c$  denote the value  $v_i(\overrightarrow{x_{w_{i-1}}})$ . Since

$$v_{i-1} + t_{i-1} \xrightarrow{w_{i-1}} v_i \quad (2)$$

we know that

$$(v_{i-1} + t_{i-1})[\overrightarrow{x_{w_{i-1}}} := c] \models \psi \quad (3)$$

where  $\psi$  is the guard of the edge responsible for  $v_{i-1} + t_{i-1} \xrightarrow{w_{i-1}} v_i$  and that

$$v_i = (v_{i-1} + t_{i-1})[\overrightarrow{x_{w_{i-1}}} := c, \overleftarrow{x_{w_{i-1}}} := 0]. \quad (4)$$

Next, we let  $v'_{i-1}$  be a valuation and  $\bar{t}_{i-1}$  be a time delay s.t.  $v'_{i-1} \in \bar{v}_{i-1} +_w t_{i-1}$  and

$$v'_{i-1} \approx_{cmax} (v_{i-1} + \bar{t}_{i-1}). \quad (5)$$

Such  $v'_{i-1}$  and  $\bar{t}_{i-1}$  are guaranteed to exist by Lemma 2:  $\bar{v}_{i-1} \in r_{i-1}$  by induction hypothesis and  $v_{i-1} \in r_{i-1}$  by construction, hence  $\bar{v}_{i-1} \approx_{cmax} \bar{v}_{i-1}$ . Then, we let

$$\bar{v}_i = v'_{i-1}[\overrightarrow{x_{w_{i-1}}} := c, \overleftarrow{x_{w_{i-1}}} := 0] \quad (6)$$

Let us check that  $v'_{i-1} \xrightarrow{w_i} \bar{v}_i$ . By (5),  $v'_{i-1}$  and  $v_{i-1} + t_{i-1}$  are equivalent. Hence,  $v'_{i-1}[\overrightarrow{x_{w_{i-1}}} := c] \approx_{cmax} (v_{i-1} + t_{i-1})[\overrightarrow{x_{w_{i-1}}} := c]$ . Thus, by (3),  $v'_{i-1}[\overrightarrow{x_{w_{i-1}}} := c] \models \psi$ , and the same transition can be fired from  $v'_{i-1}$ , leading to  $\bar{v}_i$ , by (6). Finally, by (4), (6) and (5), we deduce that  $\bar{v}_i \approx_{cmax} v_i \in r_i$ , hence  $\bar{v}_i \in r_i$ .

By construction,  $(q_0, \bar{v}_0) \xrightarrow{\bar{t}_0, w_0} (q_1, \bar{v}_1) \xrightarrow{\bar{t}_1, w_1} \dots (q_n, \bar{v}_n)$  is an accepting weak run of  $A$  on  $\theta$  with  $\text{Untime}(\theta) = w$ . Thus,  $\text{wL}(\text{RegAut}_{\exists}(A)) \subseteq \text{Untime}(\mathbb{L}(A))$ . Since  $\text{wL}(\text{RegAut}_{\exists}(A)) = L(\text{RegAut}_{\exists}(A))$ , by Proposition 2, we have  $L(\text{RegAut}_{\exists}(A)) \subseteq \text{Untime}(L(A))$ .  $\square$

**Size of the existential region automaton** The number of Alur-Dill regions on  $n$  clocks and with maximal constant  $cmax$  is at most  $R(n, cmax) = n! \times 2^n \times (2 \times cmax + 2)^n$  [2]. Adapting this result to take into account the  $\perp$  value, we have:  $|\text{Reg}(\mathbb{C}_{\Sigma}, cmax)| \leq R(2 \times |\Sigma|, cmax + 1)$ . Hence, the number of locations of  $\text{RegAut}_{\exists}(A)$  for an ECA  $A$  with  $m$  locations and alphabet  $\Sigma$  is at most  $m \times R(2 \times |\Sigma|, cmax + 1)$ . In [3], a technique is given to obtain a finite automaton recognizing  $\text{Untime}(L(A))$  for all ECA  $A$ : first transform  $A$  into a non-deterministic timed automaton [2]  $A'$  s.t.  $L(A') = L(A)$ , then compute the region automaton of  $A'$ . However, building  $A'$  incurs a blow up in the number of clocks and locations, and the size of the region automaton of  $A'$  is at most  $m \times 2^K \times R(K, cmax)$  where  $K = 6 \times |\Sigma| \times (cmax + 2)$  is an upper bound on the number of atomic clock constraints in  $A$ . Our construction thus yields a smaller automaton.

## 5 Zones and event-clocks

In the setting of timed automata, the *zone datastructure* [10] has been introduced as an effective way to improve the running time and memory consumption of on-the-fly algorithms for checking emptiness. In this section, we *adapt* this notion to the framework of ECA, and discuss forward and backward analysis algorithms. Roughly speaking, a *zone* is a symbolic representation for a set of clock valuations that are defined by constraints of the form  $x - y \prec c$ , where  $x, y$  are clocks,  $\prec$  is either  $<$  or  $\leq$ , and  $c$  is an integer constant. Keeping the difference between clock values makes sense in the setting of timed automata as all the clocks have always real values and the difference between two clock values is an invariant over the elapsing of time. To adapt the notion of zone to ECA, we need to overcome two difficulties. First, prophecy and history clocks evolve in different directions with time elapsing. Hence, it is not always the case that if  $v(x) - v(y) = c$  then  $(v + t)(x) - (v + t)(y) = c$  for all  $t$  (for instance if  $x$  is a prophecy clock and  $y$  an history clock). However, the *sum* of clocks of different types is now an invariant, so event clock zones must be definable, either by constraints of the form  $x - y \prec c$ , if  $x$  and  $y$  are both history or both prophecy clocks,

or by constraints of the form  $x + y \prec c$  otherwise. Second, clocks can now take the special value  $\perp$ . Formally, we introduce the notion of event-zone as follows.

**Definition 2.** For a set  $C$  of clocks over an alphabet  $\Sigma$ , an event-zone is a subset of  $\mathcal{V}(C)$  that is defined by a conjunction of constraints of the form  $x = \perp$ ;  $x \sim c$ ;  $x_1 - x_2 \sim c$  if  $x_1, x_2 \in \mathbb{H}_\Sigma$  or  $x_1, x_2 \in \mathbb{P}_\Sigma$ ; and  $x_1 + x_2 \sim c$  if either  $x_1 \in \mathbb{H}_\Sigma$  and  $x_2 \in \mathbb{P}_\Sigma$  or  $x_1 \in \mathbb{P}_\Sigma$  and  $x_2 \in \mathbb{H}_\Sigma$ , with  $x, x_1, x_2 \in C$ ,  $\sim \in \{\leq, \geq, <, >\}$  and  $c \in \mathbb{Z}$ .

**Event-clock Difference Bound Matrices** In the context of timed automata, Difference Bound Matrices (DBMs for short) have been introduced to represent and manipulate zones [5, 10]. Let us now adapt DBMs to event clocks. In order to adapt DBMs to event-zones, we need to be able to (i) encode constraints of the form  $x + y \prec c$  and of the form  $x' - y' \prec c$ , depending on the types of  $x, y, x'$  and  $y'$ , (ii) encode constraints of the form  $x = \perp$ , and (iii) encode the fact that a variable is not constrained by the zone. Indeed, in a DBM, this is encoded by the pair of constraints  $x \geq 0$  and  $x < +\infty$ . This is not sound in our case since  $0 \leq x < +\infty$  implies that  $x \neq \perp$ . Thus, we introduce a special symbol  $?$  to denote the absence of constraint.

Formally, an EDBM  $M$  of the set of clocks  $C = \{x_1, \dots, x_n\}$  is a  $(n+1)$  square matrix of elements from  $(\mathbb{Z} \times \{<, \leq\}) \cup \{(\infty, <), (\perp, =), (? , =)\}$  s.t. for all  $0 \leq i, j \leq n$ :  $m_{i,j} = (\perp, =)$  implies  $i = 0$  or  $j = 0$  (i.e.,  $\perp$  can only appear in the first position of a row or column). Thus, a constraint of the form  $x_i = \perp$  will be encoded with either  $m_{i,0} = (\perp, =)$  or  $m_{0,i} = (\perp, =)$ . As in the case of DBMs, we assume that the extra clock  $x_0$  is always equal to zero. Moreover, since prophecy clocks decrease with time evolving, they are encoded by their *opposite value* in the matrix. Hence the EDBM naturally encodes *sums* of variables when the two clocks are of different types. Each element  $(m_{ij}, \prec_{ij})$  of the matrix thus represents either the constraint  $x_i - x_j \prec_{ij} m_{ij}$  or the constraint  $x_i + x_j \prec_{ij} m_{ij}$ , depending on the type of  $x_i$  and  $x_j$ . Finally, the special symbol  $?$  encodes the fact that the variable is not constrained (it can take any real value, or the  $\perp$  value). Formally, an EDBM  $M$  on set of clocks  $C = \{x_1, \dots, x_n\}$  represents the zone  $\llbracket M \rrbracket$  on set of clocks  $C$  s.t.  $v \in \llbracket M \rrbracket$  iff for all  $0 \leq i, j \leq n$ : **if**  $M_{i,j} = (c, \prec)$  with  $c \neq ?$  **then**  $v^\pm(x_i) - v^\pm(x_j) \prec c$  (assuming  $v^\pm(x_0)$  denotes the value 0 and assuming that for all  $k \in \mathbb{Z} \cup \{\perp\}$ :  $\perp + k = \perp - k = k + \perp = k - \perp = \perp$ ). When  $\llbracket M \rrbracket = \emptyset$ , we say that  $M$  is *empty*. In the sequel, we also rely on the  $\leq$  ordering on EDBM elements. We let  $(m; \prec) \leq (m'; \prec')$  iff one of the following holds: either (i)  $m' = ?$ ; or (ii)  $m, m' \in \mathbb{Z} \cup \{\infty\}$  and  $m < m'$ ; or (iii)  $m = m'$  and either  $\prec = \prec'$  or  $\prec' = \leq$ .

As an example, consider the two following EDBMs that both represent  $x_1 = \perp \wedge 0 < x_3 - x_4 < 1 \wedge x_2 + x_4 \leq 2$  (where  $x_1, x_2$  are prophecy clocks, and  $x_3, x_4$  are history clocks):

$$\begin{pmatrix} (0, \leq) & (\perp, =) & (? , =) & (? , =) & (? , =) \\ (\perp, =) & (? , =) & (? , =) & (? , =) & (? , =) \\ (0, \leq) & (? , =) & (0, \leq) & (? , =) & (? , =) \\ (? , =) & (? , =) & (? , =) & (0, \leq) & (1, <) \\ (? , =) & (? , =) & (2, \leq) & (0, <) & (0, \leq) \end{pmatrix}$$

$$\begin{pmatrix} (0, \leq) & (\perp, =) & (\infty, <) & (0, \leq) & (0, \leq) \\ (\perp, =) & (?, =) & (?, =) & (?, =) & (?, =) \\ (0, \leq) & (?, =) & (0, \leq) & (0, \leq) & (0, \leq) \\ (\infty, <) & (?, =) & (\infty, <) & (0, \leq) & (1, <) \\ (\infty, <) & (?, =) & (2, \leq) & (0, <) & (0, \leq) \end{pmatrix}$$

**Normal form EDBMs** As in the case of DBMs, we define a *normal form* for EDBM, and show how to turn any EDBM  $M$  into a normal form EDBM  $M'$  s.t.  $\llbracket M \rrbracket = \llbracket M' \rrbracket$ . A non-empty EDBM  $M$  is in *normal form* iff the following holds: (i) for all  $1 \leq i \leq n$ :  $M_{i,0} = (\perp, =)$  iff  $M_{0,i} = (\perp, =)$  and  $M_{i,0} = (?, =)$  iff  $M_{0,i} = (?, =)$ , (ii) for all  $1 \leq i \leq n$ :  $M_{i,0} \in \{(\perp, =), (?, =)\}$  implies  $M_{i,j} = M_{j,i} = (?, =)$  for all  $1 \leq j \leq n$ , (iii) for all  $1 \leq i, j \leq n$ :  $M_{i,j} = (?, =)$  iff either  $M_{i,0} \in \{ (?, =), (\perp, =) \}$  or  $M_{j,0} \in \{ (?, =), (\perp, =) \}$  and (iv) the matrix  $M'$  is a *normal form DBM* [10], where  $M'$  is obtained by projecting away all lines  $1 \leq i \leq n$  s.t.  $M_{i,0} \in \{ (?, =), (\perp, =) \}$  and all columns  $1 \leq j \leq n$  s.t.  $M_{0,j} \in \{ (?, =), (\perp, =) \}$  from  $M$ . To canonically represent the empty zone, we select a particular EDBM  $M_\emptyset$  s.t.  $\llbracket M_\emptyset \rrbracket = \emptyset$ . For example, the latter EDBM of the above example is in normal form.

Then, given an EDBM  $M$ , Algorithm 1 allows to compute a normal form EDBM  $M'$  s.t.  $\llbracket M \rrbracket = \llbracket M' \rrbracket$ . This algorithm relies on the function `DBMNormalise( $M, S$ )`, where  $M$  is an  $(\ell+1) \times (\ell+1)$  EDBM, and  $S \subseteq \{0, \dots, \ell\}$ . `DBMNormalise( $M, S$ )` applies the classical normalisation algorithm for DBMs [10] on the DBM obtained by projecting away from  $M$  all the lines and columns  $i \notin S$ . Algorithm 1 proceeds in three steps. In the first loop, we look for lines (resp. columns)  $i$  s.t.  $M_{i,0}$  (resp.  $M_{0,i}$ ) is  $(\perp, =)$ , meaning that there is a constraint imposing that  $x_i = \perp$ . In this case, the corresponding  $M_{0,i}$  (resp.  $M_{i,0}$ ) must be equal to  $(\perp, =)$  too, and all the other elements in the  $i$ th line and column must contain  $(?, =)$ . If we find a  $j$  s.t.  $M_{i,j} \neq (?, =)$  or  $M_{j,i} \neq (?, =)$ , then the zone is empty, and we return  $M_\emptyset$ . Then, in the second loop, the algorithm looks for lines (resp. columns)  $i$  with the first element equal to  $(?, =)$  but containing a constraint of the form  $(c, <)$ , which imposes that the variable  $i$  must be different from  $\perp$ . We record this information by replacing the  $(?, =)$  in  $M_{i,0}$  (resp.  $M_{0,i}$ ) by the weakest possible constraint that forces  $x_i$  to have a value different from  $\perp$ . This is either  $(0, \leq)$  or  $(\infty, <)$ , depending on the type of  $x_i$  and is taken care by the `SetCst()` function. At this point the set  $S$  contains the indices of all variables that are constrained to be real. The algorithm finishes by calling the normalisation function for DBMs. Remark, in particular, that the algorithm returns  $M_\emptyset$  iff  $M$  is empty which also provides us with a test for EDBM emptiness.

**Proposition 3.** *For all EDBM  $M$ , `EDBMNormalise( $M$ )` returns a normal form EDBM  $M'$  s.t.  $\llbracket M' \rrbracket = \llbracket M \rrbracket$ .*

**Operations on zones** The four basic operations we need to perform on event-zones are: (i) *future* of an event-zone  $Z$ :  $\overrightarrow{Z} = \{v \in \mathcal{V}(\mathbb{C}_\Sigma) \mid \exists v' \in Z, t \in \mathbb{R}^{\geq 0} : v = v' + t\}$ ; (ii) *past* of an event-zone  $Z$ :  $\overleftarrow{Z} = \{v \in \mathcal{V}(\mathbb{C}_\Sigma) \mid \exists t \in \mathbb{R}^{\geq 0} : v + t \in Z\}$ ; (iii) *intersection* of two event-zones  $Z$  and  $Z'$ ; and (iv) *release* of a clock  $x$  in  $Z$ :

```

1 EDBMNormalise( $M$ ) begin
2   Let  $S = \{0\}$ ;
3   foreach  $1 \leq i \leq n$  s.t.  $M_{i,0} = (\perp, =)$  or  $M_{0,i} = (\perp, =)$  do
4     if  $\exists 1 \leq j \leq n$  s.t.  $M_{i,j} \neq (?, =)$  or  $M_{j,i} \neq (?, =)$  then return  $M_\emptyset$ ;
5      $M_{i,0} \leftarrow (\perp, =)$ ;  $M_{0,i} \leftarrow (\perp, =)$ ;
6   foreach  $0 \leq i, j \leq n$  s.t.  $M_{i,j} \notin \{ (?, =), (\perp, =) \}$  do
7      $S \leftarrow S \cup \{i, j\}$ ;
8   foreach  $i, j \in S$  do SetCst( $M_{i,j}$ ) ;
9    $M' \leftarrow$  DBMNormalise( $M, S$ ) ;
10  if  $M' = \text{Empty}$  then return  $M_\emptyset$  ;
11  return  $M'$  ;

12 SetCst( $M_{i,j}$ ) begin
13   if  $M_{i,j} = (?, =)$  then
14     if  $x_i \in \mathbb{P}_\Sigma$  and  $(x_j \in \mathbb{H}_\Sigma$  or  $x_j = x_0)$  then  $M_{i,j} \leftarrow (0, \leq)$  ;
15     else  $M_{i,j} \leftarrow (\infty, <)$  ;

```

**Algorithm 1:** A normalisation algorithm for EDBMs.

$\text{rel}_x(Z) = \{v[x := d] \mid v \in Z, d \in \mathbb{R}^{\geq 0} \cup \{\perp\}\}$ . Moreover, we also need to be able to test for inclusion of two zones encoded as EDBMs. Let  $M, M_1$  and  $M_2$  be EDBMs in normal form, on  $n$  clocks. Then:

**Future** If  $M = M_\emptyset$ , we let  $\vec{M} = M_\emptyset$ . Otherwise, we let  $\vec{M}$  be s.t.:

$$\vec{M}_{i,j} = \begin{cases} (0, \leq) & \text{if } M_{i,j} \notin \{(\perp, =), (?, =)\}, j = 0 \text{ and } x_i \in \mathbb{P}_\Sigma \\ (\infty, <) & \text{if } M_{i,j} \notin \{(\perp, =), (?, =)\}, j = 0 \text{ and } x_i \in \mathbb{H}_\Sigma \\ M_{i,j} & \text{otherwise} \end{cases}$$

**Past** If  $M = M_\emptyset$ , we let  $\overleftarrow{M} = M_\emptyset$ . Otherwise, we let  $\overleftarrow{M}$  be s.t. for all  $i, j$ :

$$\overleftarrow{M}_{i,j} = \begin{cases} (\infty, <) & \text{if } M_{i,j} \notin \{(\perp, =), (?, =)\}, i = 0 \text{ and } x_j \in \mathbb{P}_\Sigma \\ (0, \leq) & \text{if } M_{i,j} \notin \{(\perp, =), (?, =)\}, i = 0 \text{ and } x_j \in \mathbb{H}_\Sigma \\ M_{i,j} & \text{otherwise} \end{cases}$$

**Intersection** We consider several cases. If  $M^1 = M_\emptyset$  or  $M^2 = M_\emptyset$ , we let  $M^1 \cap M^2 = M_\emptyset$ . If there are  $0 \leq i, j \leq n$  s.t.  $M_{i,j}^1 \not\leq M_{i,j}^2$  and  $M_{i,j}^2 \not\leq M_{i,j}^1$ , we let  $M^1 \cap M^2 = M_\emptyset$  too. Otherwise, we let  $M^1 \cap M^2$  be the EDBM  $M'$  s.t for all  $i, j$ :  $M'_{i,j} = \min(M_{i,j}^1, M_{i,j}^2)$ .

**Release** Let  $x$  be an event clock. In the case where  $M = M_\emptyset$ , we let  $\text{rel}_x(M) = M_\emptyset$ . Otherwise, we let  $\text{rel}_x(M)$  be the EDBM s.t. for all  $i, j$ :

$$\text{rel}_x(M)_{i,j} = \begin{cases} M_{i,j} & \text{if } x_i \neq x \text{ and } x_j \neq x \\ (?, =) & \text{otherwise} \end{cases}$$



**Inclusion** We note  $M^1 \subseteq M^2$  iff  $M_{i,j}^1 \leq M_{i,j}^2$  for all  $0 \leq i, j \leq n$ .

**Proposition 4.** Let  $M, M^1, M^2$  be EDBMs in normal form, on set of clocks  $C$ . Then,  
 (i)  $\overrightarrow{\llbracket M \rrbracket} = \llbracket \overrightarrow{M} \rrbracket$ , (ii)  $\overleftarrow{\llbracket M \rrbracket} = \llbracket \overleftarrow{M} \rrbracket$ , (iii)  $\llbracket M^1 \cap M^2 \rrbracket = \llbracket M^1 \rrbracket \cap \llbracket M^2 \rrbracket$ , (iv)  
 for all clock  $x \in C$ ,  $\text{rel}_x(\llbracket M \rrbracket) = \llbracket \text{rel}_x(M) \rrbracket$  and (v)  $\llbracket M^1 \rrbracket \subseteq \llbracket M^2 \rrbracket$  iff  $M^1 \subseteq M^2$ .

*Proof.* 1. In the case where  $M = M_\emptyset$  the proof is trivial. Otherwise,  $M$  is non-empty, since it is in normal form. We assume that  $M$  is an EDBM on set of clocks  $C = \{x_1, \dots, x_n\}$ , that for all  $0 \leq i, j \leq n$ :  $M_{i,j} = (m_{i,j}, \prec_{i,j})$  and that  $\overrightarrow{M} = (m'_{i,j}, \prec'_{i,j})$ . It is easy to see that any  $v \in \llbracket \overrightarrow{M} \rrbracket$  satisfies the constraints of  $\llbracket \overrightarrow{M} \rrbracket$ . Thus,  $\llbracket \overrightarrow{M} \rrbracket \subseteq \llbracket \overrightarrow{M} \rrbracket$ .

Consider now a valuation  $v \in \llbracket \overrightarrow{M} \rrbracket$ . We need to find a delay  $t \in \mathbb{R}^{\geq 0}$  such that there exists  $v_M \in \llbracket M \rrbracket$  such that  $v_M + t = v$ . This amounts to solving the following system of inequalities:

$$\begin{cases} -m_{i0} - v(x_i) \prec_{i0} t \prec_{0i} m_{0i} - v(x_i) & \text{for all } x_i \in \mathbb{P}_\Sigma \cap C \text{ such that } m_{0i} \notin \{\perp, ?\} \\ v(x_i) - m_{i0} \prec_{i0} t \prec_{0i} v(x_i) + m_{0i} & \text{for all } x_i \in \mathbb{H}_\Sigma \cap C \text{ such that } m_{0i} \notin \{\perp, ?\} \\ 0 \leq t \end{cases}$$

with the convention that  $\infty + c = \infty - c = \infty$  and that  $-\infty + c = -\infty - c = -\infty$  for all  $c \in \mathbb{N}$ . We show that the set of solutions is not empty, i.e. that all inequalities are pairwise coherent.

Since for all  $x_i \in \mathbb{P}_\Sigma \cap C$ ,  $(m'_{0i}, \prec'_{0i}) = (m_{0i}, \prec_{0i})$ , we know that  $v(x_i) \prec_{0i} m_{0i}$  and since for all  $x_i \in \mathbb{H}_\Sigma \cap C$ ,  $(m'_{0i}, \prec'_{0i}) = (m_{0i}, \prec_{0i})$ , we also know that  $-m_{0i} \prec_{0i} v(x_i)$ . Then, none of the inequalities forces  $t$  to be negative.

Let now  $x_i, x_j$  be two prophecy clocks s.t.  $m_{0,i} \notin \{\perp, ?\}$  and  $m_{0,j} \notin \{\perp, ?\}$ . For all  $v_M \in \llbracket M \rrbracket$ ,  $-m_{i0} \prec_{i0} v_M(x_i) \prec_{0i} m_{0i}$ , and  $-m_{j0} \prec_{j0} v_M(x_j) \prec_{0j} m_{0j}$ , then  $-m_{i0} - m_{0j} \prec_1 v_M(x_i) - v_M(x_j) \prec_2 m_{0i} + m_{j0}$ , where  $\prec_1 = \leq$  iff  $\prec_{i0} = \leq$  and  $\prec_{0j} = \leq$  and  $\prec_2 = \leq$  iff  $\prec_{0i} = \leq$  and  $\prec_{j0} = \leq$ . Since  $M$  is in normal form,  $(m_{ji}, \prec_{ji}) \leq (m_{0i} + m_{j0}, \prec_2)$  and  $(m_{ij}, \prec_{ij}) \leq (m_{i0} + m_{0j}, \prec_1)$ . Since  $(m'_{ij}, \prec'_{ij}) = (m_{ij}, \prec_{ij})$  and  $(m'_{ji}, \prec'_{ji}) = (m_{ji}, \prec_{ji})$ , we deduce that  $-m_{i0} - m_{0j} \prec_1 v(x_i) - v(x_j) \prec_2 m_{0i} + m_{j0}$ . Hence,  $-m_{i0} - v(x_i) \prec_1 m_{0j} - v(x_j)$  and  $-m_{j0} - v(x_j) \prec_2 m_{0i} - v(x_i)$ . Then the constraints on  $t$  deduced from  $x_i$  and  $x_j$  are coherent. With the same arguments, we obtain that the constraints on  $t$  deduced from  $x_i, x_j \in \mathbb{H}_\Sigma \cap C$  are coherent too.

Consider now  $x_i \in \mathbb{P}_\Sigma \cap C$  and  $x_j \in \mathbb{H}_\Sigma \cap C$ . Then again, since any valuation  $v_M$  in  $\llbracket M \rrbracket$  satisfies  $-m_{i0} - m_{0j} \prec_1 v_M(x_i) + v_M(x_j) \prec_2 m_{0i} + m_{j0}$ , so does  $v$ , and one can deduce that  $-m_{i0} - v(x_i) \prec_1 v(x_j) + m_{0j}$  and  $v(x_j) - m_{j0} \prec_2 m_{0i} - v(x_i)$  and hence that the constraints on  $t$  derived from  $x_i \in \mathbb{P}_\Sigma \cap C$  and  $x_j \in \mathbb{H}_\Sigma \cap C$  are coherent.

Then, the set of solutions of the inequalities is not empty. Let  $t$  be such a solution. We let  $v_M$  be the valuation s.t.  $v_M(x) = v(x) + t$  for any  $x \in \mathbb{P}_\Sigma \cap C$  and  $v_M(x) = v(x) - t$  for all  $x \in \mathbb{H}_\Sigma \cap C$ . Such a valuation exists, and is in  $\llbracket M \rrbracket$ .

by construction. Then, since  $v = v_M + t$  with  $v_M \in \llbracket M \rrbracket$  and some  $t \in \mathbb{R}^{\geq 0}$  we deduce that  $v \in \overrightarrow{\llbracket M \rrbracket}$ . We conclude that  $\overrightarrow{\llbracket \overrightarrow{M} \rrbracket} \subseteq \overrightarrow{\llbracket M \rrbracket}$ .

2. As prophecy and history clocks evolve in opposite directions, the arguments of the proof for  $\overrightarrow{M}$  can be adapted.
3. In the case where  $M^1 = M_\emptyset$  or  $M^2 = M_\emptyset$  the proof is trivial. Otherwise,  $M^1$  and  $M^2$  are non-empty, since they are in normal form. First consider the case where there are  $0 \leq i, j \leq n$  s.t.  $M_{i,j}^1 \not\leq M_{i,j}^2$  and  $M_{i,j}^2 \not\leq M_{i,j}^1$ . By definition of  $\leq$ , this implies that either  $M_{i,j}^1$  or  $M_{i,j}^2$  is equal to  $(\perp, =)$ , and that the other constraint is of the form  $(\prec, m)$ , with  $m \in \mathbb{R}^{\geq 0} \cup \{\infty\}$ . Then, clearly  $\llbracket M^1 \rrbracket \cap \llbracket M^2 \rrbracket = \emptyset$  and thus  $\llbracket M^1 \rrbracket \cap \llbracket M^2 \rrbracket = \llbracket M_\emptyset \rrbracket = \llbracket M^1 \cap M^2 \rrbracket$ .  
Thus, let us assume that for all  $0 \leq i, j \leq n$ ,  $\min\{M_{i,j}^1, M_{i,j}^2\}$  is defined. Let  $v$  be a valuations on the set of clocks  $C = \{x_1, \dots, x_n\}$ , let  $M$  be an EDBM on  $C$ . Then for all  $0 \leq i, j \leq n$ , we say that  $v$  satisfies  $M_{i,j} = (m_{i,j}, \prec_{i,j})$  (denoted  $v \models M_{i,j} = (m_{i,j}, \prec_{i,j})$ ) iff:

- (a) either  $m_{i,j} = ?$
- (b) or  $i = 0$  and  $m_{i,j} = v(x_j) = \perp$
- (c) or  $j = 0$  and  $m_{i,j} = v(x_i) = \perp$
- (d) or  $m_{i,j} \notin \{?, \perp\}$  and  $|x_i| - |x_j| \prec_{i,j} m_{i,j}$ , assuming  $\perp + c = c + \perp = \perp - c = c - \perp = \perp$  for all  $c$ .

Then, clearly,  $\llbracket M \rrbracket = \{v \mid \forall 0 \leq i, j \leq n : v \models M_{i,j}\}$ .

Then observe that, by definition of the ordering  $\leq$  on EDBM constraints:

$$(v \models (m_1, \prec_1) \text{ and } v \models (m_2, \prec_2)) \quad \text{iff} \quad v \models \min\{(m_1, \prec_1), (m_2, \prec_2)\}$$

By definition of  $M^1 \cap M^2$ , we conclude that  $\llbracket M^1 \rrbracket \cap \llbracket M^2 \rrbracket = \llbracket M^1 \cap M^2 \rrbracket$ .

4. In the case where  $M = M_\emptyset$  the proof is trivial. Otherwise,  $M$  is non-empty, since it is in normal form. Let us assume that  $x$  is the clock of index  $k$  in  $C$ . We first examine the case where  $M_{k0} = (?, =)$ , then  $\text{rel}_x(M) = M$  since  $M$  is in normal form. Since  $x$  is already unconstrained in  $M$ , we have  $\text{rel}_x(\llbracket M \rrbracket) = \llbracket M \rrbracket$ . Hence  $\text{rel}_x(\llbracket M \rrbracket) = \llbracket M \rrbracket = \llbracket \text{rel}_x(M) \rrbracket$ .

Otherwise, let us assume that  $C = \{x_1, \dots, x_n\}$  and that for all  $0 \leq i, j \leq n$ :  $M_{i,j} = (m_{i,j}, \prec_{i,j})$ . Let  $v \in \text{rel}_x(\llbracket M \rrbracket)$ . Then there is some  $v' \in \llbracket M \rrbracket$ , such that  $v'(y) = v(y)$ , for all clock  $y \neq x$  in  $C$ . Since  $v'$  satisfies all the constraints of  $M$ ,  $v$  satisfies all the constraints of  $\llbracket M \rrbracket$  related to clocks different from  $x$ , and hence  $v \in \llbracket \text{rel}_x(M) \rrbracket$ . Thus,  $\text{rel}_x(\llbracket M \rrbracket) \subseteq \llbracket \text{rel}_x(M) \rrbracket$ .

Conversely, let  $v \in \llbracket \text{rel}_x(M) \rrbracket$ . We consider two cases. Either  $M_{k0} = (\perp, =)$ . We let  $v'$  be the valuation s.t.  $v'(x) = \perp$  and for all  $y \neq x$ :  $v'(y) = v(y)$ . Clearly  $v' \in \llbracket M \rrbracket$  since  $M$  is non-empty and in normal form. Hence,  $v \in \text{rel}_x \llbracket M \rrbracket$ . Otherwise  $M_{k0} = (m, \prec)$  with  $m \in \mathbb{R}^{\geq 0} \cup \{\infty\}$ , since we have already ruled

out the case  $M_{k0} = (?, =)$ . We let  $v'$  be a valuation that is a solution of the following set of inequalities if  $x$  is an history clock:

$$\begin{aligned} v'(y) &= v(y) && \text{for all } y \neq x \\ -m_{0k} &\prec_{0k} v'(x) && \prec_{k0} m_{k0} \\ -m_{jk} &\prec_{jk} v'(x) - v'(x_j) && \prec_{kj} m_{kj} && \text{for all } x_j \in (\mathbb{H}_\Sigma \cap C) \setminus \{x\} \\ -m_{jk} &\prec_{jk} v'(x) + v'(x_j) && \prec_{kj} m_{kj} && \text{for all } x_j \in (\mathbb{P}_\Sigma \cap C) \setminus \{x\} \end{aligned}$$

or a solution of the following set of inequalities if  $x$  is a prophecy clock:

$$\begin{aligned} v'(y) &= v(y) && \text{for all } y \neq x \\ -m_{k0} &\prec_{k0} v'(x) && \prec_{0k} m_{0k} \\ -m_{kj} &\prec_{kj} v'(x) - v'(x_j) && \prec_{jk} m_{jk} && \text{for all } x_j \in (\mathbb{P}_\Sigma \cap C) \setminus \{x\} \\ -m_{jk} &\prec_{jk} v'(x) + v'(x_j) && \prec_{kj} m_{kj} && \text{for all } x_j \in (\mathbb{H}_\Sigma \cap C) \setminus \{x\} \end{aligned}$$

assuming as usual that  $\perp + c = c + \perp = \perp - c = c - \perp = \perp$ .

Since  $M$  is in normal form, such a  $v'$  exists (otherwise, some of the constraints could be strengthened without modifying the zone, and  $M$  is not in normal form), and it is in  $\llbracket M \rrbracket$ . Hence  $v$  is in  $\text{rel}_x(\llbracket M \rrbracket)$ . We conclude that  $\llbracket \text{rel}_x(M) \rrbracket \subseteq \text{rel}_x(\llbracket M \rrbracket)$ .

5. The proof stems from the fact that  $\llbracket M^1 \rrbracket \subseteq \llbracket M^2 \rrbracket$  **iff**  $\llbracket M^1 \rrbracket \cap \llbracket M^2 \rrbracket = \llbracket M^1 \rrbracket$  **iff**  $\llbracket M^1 \cap M^2 \rrbracket = \llbracket M^1 \rrbracket$  **iff**,  $\min(M_{i,j}^1, M_{i,j}^2) = M_{i,j}$  for all  $0 \leq i, j \leq n$  (by Proposition 4).

□

**Forward and backward analysis** We present now the forward and backward analysis algorithms adapted to ECA. From now on, we consider an ECA  $A = \langle Q, q_i, \Sigma, \delta, \alpha \rangle$ . We also let  $\text{Post}((q, v)) = \{(q', v') \mid \exists t, a : (q, v) \xrightarrow{t,a} (q', v')\}$  and  $\text{Pre}((q, v)) = \{(q', v') \mid \exists t, a : (q', v') \xrightarrow{t,a} (q, v)\}$  and we extend those operators to sets of states in the natural way. Moreover, given a set of valuations  $Z$  and a location  $q$ , we abuse notations and denote by  $(q, Z)$  the set  $\{(q, v) \mid v \in Z\}$ . Also, we let  $\text{Post}^*((q, Z)) = \bigcup_{n \in \mathbb{N}} \text{Post}^n((q, Z))$  and  $\text{Pre}^*((q, Z)) = \bigcup_{n \in \mathbb{N}} \text{Pre}^n((q, Z))$ , where  $\text{Post}^0((q, Z)) = (q, Z)$  and  $\text{Post}^n((q, Z)) = \text{Post}(\text{Post}^{n-1}((q, Z)))$ , and similarly for  $\text{Pre}^n((q, Z))$ . The Post and Pre operators are sufficient to solve language emptiness for ECA:

**Lemma 3** (adapted from [3], Lemma 1). *Let  $A = \langle Q, q_i, \Sigma, \delta, \alpha \rangle$  be an ECA, let  $I = \{(q_i, v) \mid v \text{ is initial}\}$ , and let  $\bar{\alpha} = \{(q, v) \mid q \in \alpha \text{ and } v \text{ is final}\}$ . Then:*

$$\text{Post}^*(I) \cap \bar{\alpha} \neq \emptyset \text{ iff } \text{Pre}^*(\bar{\alpha}) \cap I \neq \emptyset \text{ iff } L(A) \neq \emptyset.$$

Let us show how to compute these operators on event-zones. Given a location  $q$ , an event-zone  $Z$  on  $\mathbb{C}_\Sigma$ , and an edge  $e = (q, a, \psi, q') \in \delta$ , we let:

$$\begin{aligned} \text{Post}_e((q_1, Z)) &= \begin{cases} \left( q', \left( \text{rel}_{\vec{x}_a}(\text{rel}_{\vec{x}_a}(\vec{Z} \cap (\vec{x}_a = 0)) \cap \psi) \right) \cap (\vec{x}_a = 0) \right) & \text{if } q_1 = q \\ \emptyset & \text{otherwise} \end{cases} \\ \text{Pre}_e((q_1, Z)) &= \begin{cases} \left( q, \overleftarrow{\left( \text{rel}_{\vec{x}_a}(\text{rel}_{\vec{x}_a}(Z \cap (\vec{x}_a = 0)) \cap \psi) \right) \cap (\vec{x}_a = 0)} \right) & \text{if } q_1 = q' \\ \emptyset & \text{otherwise} \end{cases} \end{aligned}$$

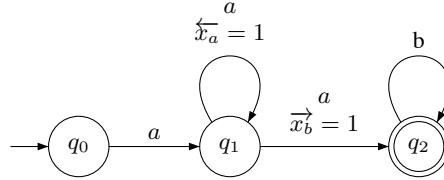
```

1 ForwExact begin
2   Let Visited =  $\emptyset$  ; Let Wait =  $\{(q_i, Z_0)\}$  ;
3   while Wait  $\neq \emptyset$  do
4     Get and remove  $(q, Z)$  from Wait ;
5     if  $q \in \alpha$  and  $Z \subseteq Z_f$  then return Yes ;
6     if there is no  $(q, Z') \in \text{Visited}$  s.t.  $Z \subseteq Z'$  then
7       Visited := Visited  $\cup \{(q, Z)\}$  ;
7       Wait := Wait  $\cup \text{Post}((q, Z))$  ;
8   return No ;

9 BackExact begin
10  Let Visited =  $\emptyset$  ; Let Wait =  $\{(q, Z_f) \mid q \in \alpha\}$  ;
11  while Wait  $\neq \emptyset$  do
12    Get and remove  $(q, Z)$  from Wait ;
13    if  $q = q_i$  and  $Z \subseteq Z_0$  then return Yes ;
14    if there is no  $(q, Z') \in \text{Visited}$  s.t.  $Z \subseteq Z'$  then
15      Visited := Visited  $\cup \{(q, Z)\}$  ;
15      Wait := Wait  $\cup \text{Pre}((q, Z))$  ;
16  return No ;

```

**Algorithm 2:** The forward and backward algorithms



**Figure 3:** An ECA for which backward analysis does not terminate.

Then, it is easy to check that  $\text{Post}((q, Z)) = \cup_{e \in \delta} \text{Post}_e((q, Z))$  and  $\text{Pre}((q, Z)) = \cup_{e \in \delta} \text{Pre}_e((q, Z))$ . With the algorithms on EDBMs presented above, these definitions can be used to compute the **Pre** and **Post** of zones using their EDBM encodings. Remark that **Pre** and **Post** return *sets of event-zones* as these are not closed under union.

Let us now consider the **ForwExact** and **BackExact** algorithms to test for language emptiness of ECA, shown in Algorithm 2. In these two algorithms  $Z_0$  denotes the zone  $\bigwedge_{x \in \mathbb{H}_\Sigma} x = \perp$  containing all the possible initial valuations and  $Z_f$  denotes the zone  $\bigwedge_{x \in \mathbb{P}_\Sigma} x = \perp$  representing all the possible final valuations. By Lemma 3, it is clear that **ForwExact** and **BackExact** are correct when they terminate. Unfortunately, Fig. 3 shows an ECA on which the backward algorithm does not terminate. Since history and prophecy clocks are symmetrical, this example can be adapted to define an ECA on which the forward algorithm does not terminate either. Remark that in the case of timed automata, the forward analysis is not guaranteed to terminate, whereas the backward

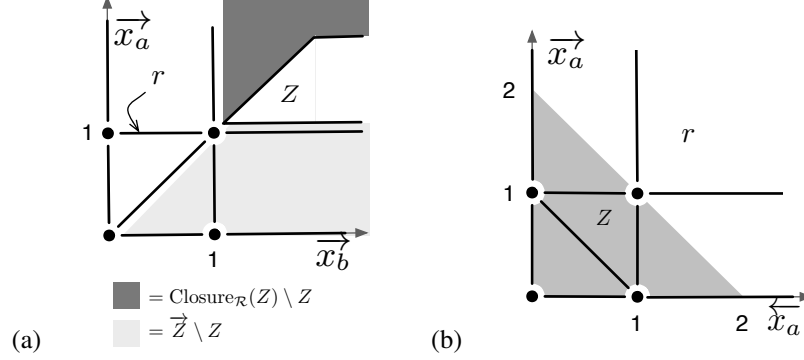


Figure 4: Examples for  $\text{Closure}_{\mathcal{R}}$  and  $\text{Approx}_k$

analysis *always terminates* (the proof relies on a bisimulation argument) [1].

**Proposition 5.** *Neither ForwExact nor BackExact terminate in general.*

*Proof.* We give the proof for **BackExact**, a similar proof for **ForwExact** can then be deduced by symmetry. Consider the ECA in Fig. 3. Running the backward analysis algorithm from  $(q_2, Z_f)$ , we obtain, after selecting the transition  $e = (q_2, b, \text{true}, q_2)$ , the zone  $Z_1 = \vec{x}_a = \perp \wedge \vec{x}_b = 0$ . Then, the transition  $e' = (q_1, a, \vec{x}_b = 1, q_2)$  is back-firable and we attain the zone  $Z_2 = \vec{x}_a \geq 0 \wedge \vec{x}_b \geq 1 \wedge \vec{x}_b - \vec{x}_a = 1$ . At this point the transition  $e'' = (q_1, a, \vec{x}_a = 1, q_1)$  is back-firable, which leads to the zone  $Z_3 = \vec{x}_b \geq 1 \wedge \vec{x}_a \geq 0 \wedge 0 \leq \vec{x}_a \leq 1 \wedge \vec{x}_b - \vec{x}_a \geq 1 \wedge \vec{x}_b + \vec{x}_a \geq 2$ . The back-firing of the  $e''$  transition can be repeated, and, by induction, after  $n$  iterations of the loop, the algorithm reaches the zone  $Z^n = \vec{x}_b \geq n \wedge \vec{x}_a \geq 0 \wedge 0 \leq \vec{x}_a \leq 1 \wedge \vec{x}_b - \vec{x}_a \geq n \wedge \vec{x}_b + \vec{x}_a \geq n + 1$ . Thus, the condition of the **if** in line 14 is always fulfilled, and the algorithm visits an infinite number of zones, without reaching  $q_0$ .  $\square$

## 6 Future work: widening operators

As said earlier, the zone-based *forward* analysis algorithm does not terminate either in the case of timed automata. To recover termination, *widening operators* have been defined. The most popular widening operator is the so-called  $k$ -approximation on zones [8]. Roughly speaking, it is defined as follows: in the definition of the zone, replace any constraint of the form  $x_i < c$  or  $x_i - x_j < c$ , by respectively  $x_i < \infty$  and  $x_i - x_j < \infty$  if and only if  $c > k$ , and replace any constraint of the form  $c < x_i$  or  $c < x_i - x_j$ , by respectively  $k < x_i$  and  $k < x_i - x_j$ , if and only if  $c > k$ . Such an operator can be easily computed on DBMs, and is a standard operation implemented in several tools such as as UppAal [4] for more more than 15 years. Nevertheless, this operator has been widely discussed in the recent literature since Bouyer has pointed out

several flaws in the proposed proofs of soundness [6]. Actually, the  $k$ -approximation is *sound* when the timed automaton contains *no diagonal constraints*. Unfortunately,  $k$ -approximation is *not sound* when the timed automaton contains diagonal constraints, and *no sound widening operator exists* in this case.

In [6], Bouyer identifies some subclasses of timed automata for which the widening operator is provably correct. The idea of the proof relies mainly on the definition of another widening operator, called the *closure by regions*, which is shown to be *sound*. The closure by regions of a zone  $Z$ , with respect to a set of regions  $\mathcal{R}$  is defined as the smallest set of regions from  $\mathcal{R}$  that have a non-empty intersection with  $Z$ , i.e.  $\text{Closure}_{\mathcal{R}}(Z) = \{r \in \mathcal{R} \mid Z \cap r \neq \emptyset\}$ . Then, the proof concludes by showing that  $\text{Approx}_k(Z)$  is sound for some values of  $k$  (that are proved to exist) s.t.

$$Z \subseteq \text{Approx}_k(Z) \subseteq \text{Closure}_{\mathcal{R}}(Z). \quad (7)$$

In the perspective of bringing ECA from theory to implementation, *provably correct* widening operators are necessary, since neither the forward nor the backward algorithm terminate in general. We plan to adapt the  $k$ -approximation to ECA, and we believe that we can follow the general idea of the proof in [6]. However, the proof techniques will not be applicable in a straightforward way, for several reasons. First, the proof of [6] relies on the following property, which holds in the case of timed automata: for all zone  $Z$  and all location  $q$ :  $\text{Post}((q, \text{Closure}_{\mathcal{R}}(Z))) \subseteq \text{Closure}_{\mathcal{R}}(\text{Post}((q, Z)))$ . Unfortunately this is not the case in general with ECA. Indeed, consider the zone  $Z$  and the region  $r$  in Fig. 4 (a). Clearly,  $r$  is included in  $\overline{\text{Closure}_{\mathcal{R}}(Z)}$  but  $r$  is not included in  $\text{Closure}_{\mathcal{R}}(\overrightarrow{Z})$  (recall that prophecy clocks decrease with time elapsing). Moreover, the definition of the  $k$  approximation will need to be adapted to the case of ECA. Indeed, the second inclusion in (7) does not hold when using the  $k$ -approximation defined for timed automata, which merely replaces all constants  $> k$  by  $\infty$  in the constraints of the zone. Indeed, consider the event-zone  $Z$  defined by  $\overleftarrow{x}_a + \overrightarrow{x}_a \leq 2$  in Fig. 4 (b), together with the set of regions  $\mathcal{R} = \text{Reg}(\mathbb{C}_{\{a\}}, 1)$ . Clearly, with such a definition, the constraint  $\overleftarrow{x}_a + \overrightarrow{x}_a \leq 2$  would be replaced by  $\overleftarrow{x}_a + \overrightarrow{x}_a < \infty$ , which yields an approximation that intersects with  $r$ , and is thus not contained in  $\text{Closure}_{\mathcal{R}}(Z)$ . We keep open for future works the definition of a provably correct adaptation of the  $k$ -approximation for ECA.

## References

- [1] R. Alur. Timed automata. In *Proceedings of CAV'99*, volume 1633 of *Lecture Notes in Computer Science*, pages 8–22. Springer, 1999.
- [2] R. Alur and D. Dill. A Theory of Timed Automata. *Theoretical Computer Science*, 126(2):183–236, 1994.
- [3] R. Alur, L. Fix, and T. A. Henzinger. Event-clock automata: a determinizable class of timed automata. *Theoretical Computer Science*, 211(1-2):253–273, 1999.

- [4] G. Behrmann, A. David, K. G. Larsen, J. Håkansson, P. Pettersson, W. Yi, and M. Hendriks. Uppaal 4.0. In *Proceedings of QEST'06*, pages 125–126. IEEE Computer Society, 2006.
- [5] R. Bellman. *Dynamic Programming*. Princeton university press, 1957.
- [6] P. Bouyer. Forward analysis of updatable timed automata. *Formal Methods in System Design*, 24(3):281–320, May 2004.
- [7] M. Bozga, C. Daws, O. Maler, A. Olivero, S. Tripakis, and S. Yovine. Kronos: A model-checking tool for real-time systems. In *Proceedings of CAV'98*, volume 1427 of *Lecture Notes in Computer Science*, pages 546–550. Springer, 1998.
- [8] C. Daws and S. Tripakis. Model checking of real-time reachability properties using abstractions. In B. Steffen, editor, *Proceedings of TACAS'98*, volume 1384 of *Lecture Notes in Computer Science*, pages 313–329. Springer, 1998.
- [9] B. Di Giampaolo, G. Geeraerts, J. Raskin, and N. Sznajder. Safraless procedures for timed specifications. In *Proceedings of FORMATS'10*, volume 6246 of *Lecture Notes in Computer Science*, pages 2–22. Springer, 2010.
- [10] D. L. Dill. Timing assumptions and verification of finite-state concurrent systems. In *Proceedings of Automatic Verification Methods for Finite State Systems*, volume 407 of *Lecture Notes in Computer Science*, pages 197–212. Springer, 1989.
- [11] C. Dima. Kleene theorems for event-clock automata. In *Proceedings of FCT'99*, volume 1684 of *Lecture Notes in Computer Science*, pages 215–225. Springer, 1999.
- [12] D. D'Souza and N. Tabareau. On timed automata with input-determined guards. In *Proceedings of FORMATS/FTRTFT'04*, volume 3253 of *Lecture Notes in Computer Science*, pages 68–83, 2004.
- [13] J.-F. Raskin and P.-Y. Schobbens. The logic of event clocks: decidability, complexity and expressiveness. *Automatica*, 34(3):247–282, 1998.
- [14] M. Sorea. Tempo: A model-checker for event-recording automata. In *Proceedings of RT-TOOLS'01*, Aalborg, Denmark, August 2001.
- [15] N. Tang and M. Ogawa. Event-clock visibly pushdown automata. In *Proceedings of SOFSEM'09*, volume 5404 of *Lecture Notes in Computer Science*, pages 558–569. Springer, 2009.